

A Modified Approach To EEG Artifact Removal Using ICA, DWT And Clustering

Pawanpreet Kaur Johal¹, Dr. Neelu Jain²

P.G. Student, Department of Electronics and Communication Engineering, PEC University of Technology, Chandigarh, India¹

Associate Professor, Department of Electronics and Communication Engineering, PEC University of Technology, Chandigarh, India²

ABSTRACT: EEG or Electroencephalography records the electrical activity occurring inside the brain due to communication that takes place between neurons which constitute the brain. It is a vital bio-potential that reflects variations in brain's intricate internal activities and hence can provide valuable understanding of the brain's working. In recent times it has been found to be useful in spotting neural ailments and also in the development of Human Machine Interfaces (HMI). However, EEG is highly vulnerable to being corrupted by non-cognitive signals whether biological or non-biological, called artifacts, which can considerably affect the end performance of an EEG based system. Several approaches have been suggested in literature to remove these artifacts-automatically as well as manually. In this paper a new system based on the combined use of ICA, DWT and Hierarchical Clustering is proposed along with the implementation of two existing techniques based on DWT+ICA and ICA+HC. Technique-1 based on ICA and DWT efficiently removes non-physiological signals while the technique-2 based on ICA and Hierarchical Clustering performs moderately well for both types of noise signals. The proposed technique is found to be very effective in removing biological artifacts and performs moderately well for non-biological signals.

KEYWORDS: EEG, ICA, DWT, Hierarchical Clustering(HC), Correlation Coefficient.

I. INTRODUCTION

Brain, the largest of all human organs, controls all its activities. Human cognitive headways and associated brain connotations are fundamentally tied both to the milieu and physical structure and actions. In order to scrutinize natural human cognition, what is requisite is the imaging of neural activity while contributors perform naturally enthused actions and interactions within a realistic environment [1]. EEG is one such system that offers advantages of low operation cost, non-invasion, high temporal resolution and better study of auditory responses as it is silent [2]. EEG represents electrical exploit carried out by neural cells in reaction to some kind of a inducement be it internal or external. As such it is an expedient bio potential that gives a decent estimate of internal functioning of the brain. Neural activity as epitomized by EEG is in the form of brain waves, ERPs (Event Related Potential), SSVEPs (Steady State Visual Evoked Potential) etc. BCIs or Brain Computer Interfaces make use of these neural activity indicators to communicate with machines without the use of muscles.

EEG has a very high temporal resolution and hence can provide output information within milliseconds of the presentation of the stimulus. However, it has a major drawback of being highly prone to being influenced by non-neural signals called artifacts. Electrodes do not discriminate between the electrical signals appearing at them and hence non-cognitive electrical action inside the body also gets logged along with the EEG. These artifacts may be physiological as well as non-physiological in foundation. Examples of these artifacts are given as follows:

Biological Artifacts: Eye blinking and eye movements (EOG), muscle movement (EMG), breathing, sweating, and heartbeat etc.

Non-Biological Artifacts: Electrode impedance, Power line noise etc.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 7, July 2016

Brain waves are most commonly used in EEG analysis. These are basically of four types and have been explained briefly in Table 1 [2, 3].

TABLE 1: MAJOR BRAIN WAVES

Name	Symbol	Frequency (Hz)	Significance
Delta	δ	0-4	Characterizes deep slumber
Theta	Θ	4-7	Normally found in young children; abnormal in awake adults.
Alpha	α	7-12	Characterizes relaxed consciousness state
Beta	β	13-30	Low amplitude β shows active thinking; rhythmic β associated with drug effects

While Delta waves predominate during deep sleep, Alpha waves signify alert state of mind. Similarly, Beta waves and Theta waves also signify specific neural states as shown in the table. Any kind of aberration or artifact in the pattern of these waves can be used to detect abnormality in functioning of the brain.

Artifactual presence in EEG is predominantly harmful for it may impact the final outcome of the EEG based system. Artifacts have a propensity to emulate cognitive and psychological activity and hence can result in faulty results [4]. As such removal of artifacts is a necessary to realize efficient BCIs. Artifact removal forms a part of signal pre-processing in commercial BCIs and a multitude of techniques have been proposed by neurologists and engineers towards this end.

II. RELATED WORK

In most of the techniques proposed in literature for removal of artifacts, the authors have made use of the fact that the artifacts are statistically independent of the causal neural activity and hence have employed ICA (Independent Component Analysis) to isolate the recorded EEG signal into independent source signal approximations which are then further treated to eliminate artifacts.

Discrete Wavelet Transform or DWT has been used along with ICA to de-noise the EEG signals. It has been used pre- as well as post- ICA decomposition of EEG signals. In GuiseppinaInusso et al. 2007 DWT was used pre-ICA decomposition to make use of the spectral information of the artifacts. However, the Wavelet Components (WCs) had to be physically identified as artifactual or not based on the investigators' prior knowledge. This method was then automated by using quantitative measures of Kurtosis and Renyi's Entropy to recognize artifacts [3]. Scholars have also experimented with Spatially Constrained ICA and DWT using Otsu's Thresholding structure to automatically eliminate artifacts and have got encouraging results [5, 6]. Wavelet-ICA method has also been further protracted by including a-priori data about artifacts to be eliminated thereby creating a fully automatic system [7]. However, a major drawback of this method is that it could only be used for such artifacts whose proper a-priori information could be found.

Another popularly used method to eradicate artifacts is clustering. K-means clustering, Fuzzy C-means Clustering and Hierarchical Clustering have all been employed to this end [8, 9, 10]. Despite the fact of all these methods reporting good results, a major drawback was observed in case of K-means and Fuzzy Clustering as both these methods required the number of clusters to be defined beforehand which was not always probable [11, 12, 13]. Due to the apparent drawbacks of these clustering methods, researchers have largely concentrated on the use of hierarchical clustering to automatically group and remove artifacts. They have used mutual information based Hierarchical Clustering (HC) to

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 7, July 2016

enhance the late positive ERPs [14]. ICA + HC have been used with a 10 dimensional feature vector to automatically remove biological artifacts and also to remove non-biological artifacts by using electrode impedance [11, 12, 13].

III. TECHNIQUES IMPLEMENTED

In this paper 2 most commonly used techniques for artifact removal have been implemented and a third technique has been devised using a combined form of the above two techniques. These techniques are described in the following parts.

A. TECHNIQUE-1: ICA+DWT

Wavelet Transform and ICA together have been proven to be very effective when used together to remove artifacts by a large number of authors [5, 6,15]. The wavelet decomposition escalates the redundancy in the dataset and as such augments the performance of artifact elimination even with a restricted number of channels. [3] The step by step procedure of the technique is given as follows:

Step-1: EEG data recorded from 10 channels is loaded into MATLAB.

Step-2: Ten channel dataset is subjected to DWT analysis using Daubechies wavelet of order four i.e.db4, thereby decomposing it into four major EEG rhythms-Delta, Theta, Alpha and Beta Wavelet Components (WCs).

Step-3: Wavelet Components associated with artifactual events are automatically identified by means kurtosis and Renyi's entropy.

Step-4: WCs chosen above are subjected to ICA so as to focus the artifactual content in a few independent components. This produces Wavelet Independent Components (WICs).

Step-5: WICs with considerable amount of artifactual content are automatically selected and rejected again using kurtosis and Renyi's entropy.

Step-6: Inverse ICA is performed to obtain cleaned WCs.

Step-7: Inverse DWT is performed on the above cleaned WCs to obtain clean EEG signals.

B. TECHNIQUE-2: ICA + HIERARCHICAL CLUSTERING

Various authors have used clustering techniques along with ICA to clean noisy EEG signals[8,9,11,13,16]. Hierarchical Clustering is the most commonly used of all the available clustering techniques because the numbers of clusters need not to be defined a priori. The step by step procedure of the technique is given as follows:

Step-1: EEG data recorded from 10 channels is loaded into MATLAB.

Step-2: ICA is performed on the above dataset to obtain Independent Components (ICs) on which further calculations are to be done.

Step-3: For each IC temporal, spatial and spectral features are calculated. Kurtosis gives the temporal feature as a measure of random peakyness while the spatial feature is given by the median of electrode positions as it helps to identify ICs with maximum contribution from possible artifact-affected electrode. The spectral feature consists of average band-power calculated for all the 4 sub-bands i.e. Delta, Theta, Alpha and Beta.

Step-4: The features calculated in the above step are used to perform hierarchical clustering of the ICs such that the ICs with maximum artifactual information are clustered into one group leaving the other clean ICs in the second cluster.

Step-5: ICs in the cluster of artifacts are rejected

Step-6: Inverse ICA is performed on the remaining clean ICs to obtain the clean EEG signals.

C. TECHNIQUE-3: ICA with combined use of DWT and HIERARCHICAL CLUSTERING

This is the new method proposed after a comprehensive study of the above two techniques. It combines the use of both DWT and Hierarchical Clustering along with ICA. The step by step procedure of the technique is given as follows:

Step-1: EEG data recorded from 10 channels is loaded into MATLAB.

Step-2: Ten channel dataset is subjected to DWT analysis using Daubechies wavelet of order 4 i.e.db4, thereby decomposing it into 4 major EEG rhythms-Delta, Theta, Alpha and Beta Wavelet Components.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 7, July 2016

Step-3: ICA is performed on the above Wavelet Components to obtain Wavelet Independent Components (WICs) on which further calculations are to be done.

Step-4: For each WIC temporal, spatial and spectral features are calculated. Kurtosis gives the temporal feature as a measure of random peakyness while the spatial feature is given by the median of electrode positions as it helps to identify ICs with maximum contribution from possible artifact-affected electrode. The spectral feature consists of average band-power calculated for all the 4 sub-bands i.e. Delta, Theta, Alpha and Beta obtained after DWT.

Step-5: The features calculated in the above step are used to perform hierarchical clustering of the ICs such that the WICs with maximum artifactual information are clustered into one group leaving the other clean WICs in the second cluster.

Step-6: WICs in the cluster of artifacts are rejected.

Step-7: Inverse ICA is performed on the remaining clean WICs to obtain the clean WCs.

Step-8: Inverse DWT is performed on the above cleaned WCs to obtain clean EEG signals.

The above three techniques were applied to a common dataset of 10 channels and observations were made.

IV. SOFTWARE AND DATA COLLECTION

The entire work has been done in MATLAB and EEGLAB version 11 has been used to do the pre-processing. Standard dataset available with EEGLAB has been used. 10-second recording from 10 channels was used. Figure 1 shows the electrode positions considered in the work.

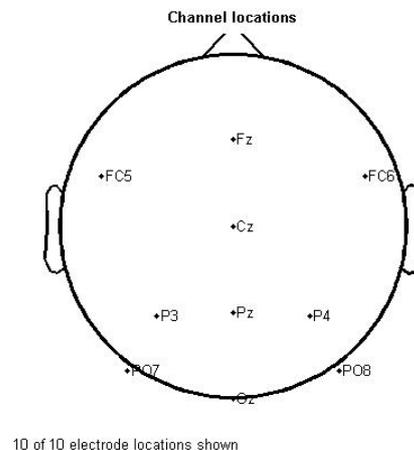


Figure 1: Channel Locations

The electrodes used are: Fz, FC5, FC6, Cz, P3, Pz, P4, PO7, Oz and PO8. These electrodes were chosen because most of the brain activity is concentrated in these areas and it is easy to record to as well.

Four artifacts have been used for the testing purpose, namely Electrical Shift Artifact, Linear Trend Artifact, EMG Artifact and Eye Blink Artifact. These artifacts have been chosen because they affect the EEG signals most frequently. These have been simulated in MATLAB. The simulated artifacts are shown in figures 2 to 5.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 7, July 2016

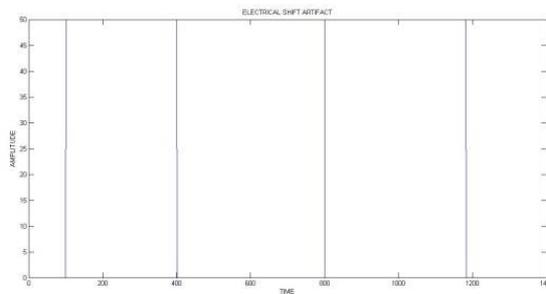


Figure 2: Electrical Shift Artifact

This artifact occurs when the reference electrode has not been properly calibrated and the electrical supply is also fluctuating.

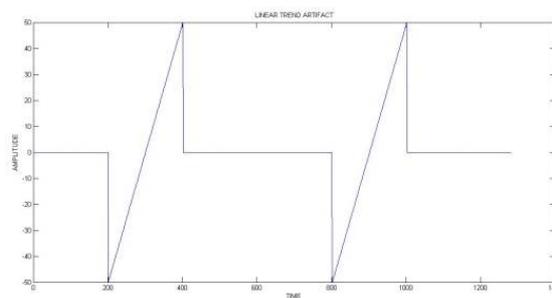


Figure 3: Linear Trend Artifact

This artifact occurs when the electrodes are not properly connected. As a result it causes the EEG signals to be shifted in amplitude.

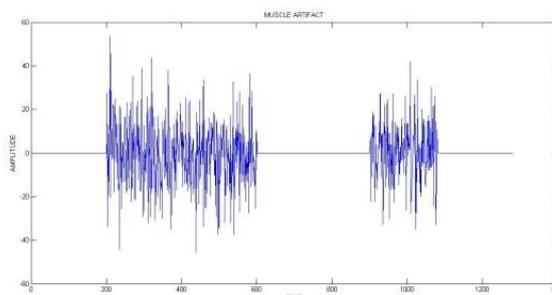


Figure 5: EMG Artifact

This artifact is generated due to muscle movement in the body. Mostly it is generated due to clenching of jaw or movement of temporal muscles or movement of eyebrows.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 7, July 2016

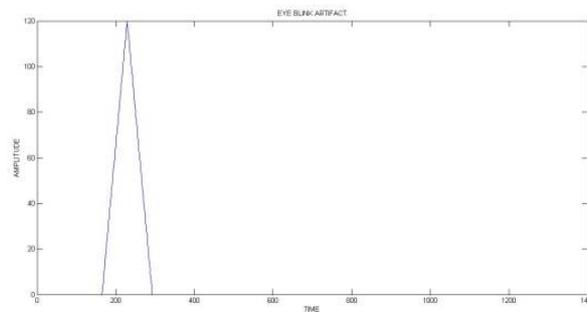


Figure 4: Eye Blink Artifact

This artifact is generated due to eye blinks by the subject.

These four artifacts were added to four channels- Fz, FC6, Pz and PO7. The figures 6 to 9 show the channels after addition of artifacts.

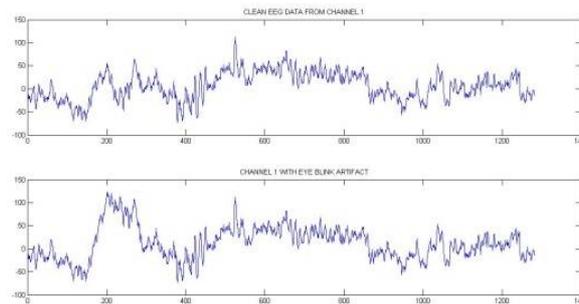


Figure 6: CHANNEL-1

Eye blink artifact was added to channel-1 or Fz. It is clearly visible as the sudden increased slope of the signal and abrupt appearance of a peak in the EEG signal.

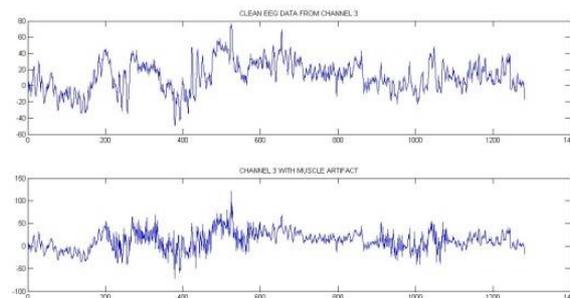


Figure 7: CHANNEL-3

EMG or muscle artifact was added to channel-3 or FC6. It can be seen as abrupt amplitude fluctuations in the above figure.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 7, July 2016

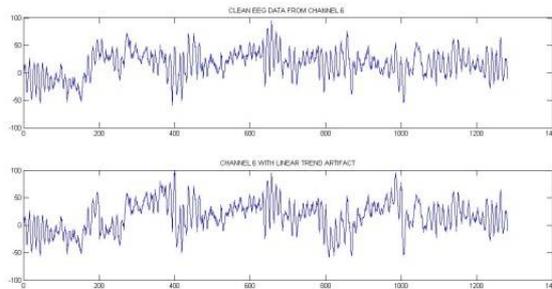


Figure 8: CHANNEL-5

Linear Trend Artifact was added to channel-5 or Pz. It is visible as enhanced slope of the EEG signal at two places.

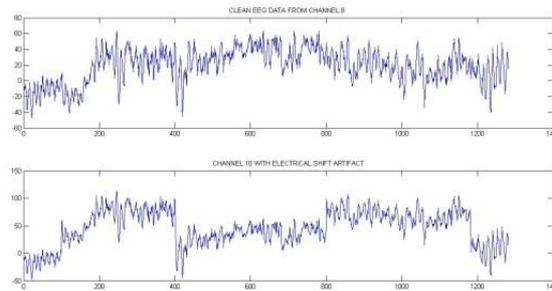


Figure 9: CHANNEL-8

Electrical Shift artifact has been added to PO7 or channel 8. This artifact abruptly shifts the signal's amplitude.

V. EXPERIMENTAL RESULTS

In order to test the results correlation was performed between the original signals and processed signals. Correlation between two signals gives the measure of degree of similarity between the signals being considered. Its value lies between 0 and 1. The higher the value the greater is the similarity and hence better the performance of the artifact removal technique. Figures 10 to 12 show the graph for correlation coefficient for all the three techniques.

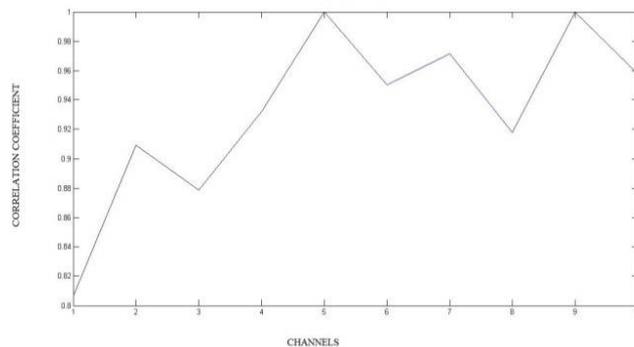


Figure 10: Correlation Coefficients for Technique-1 (ICA + DWT)

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 7, July 2016

In this figure the X-axis shows the channel number while the Y-axis shows the value of correlation coefficient. As can be seen this technique gives good results in most of the cases but is highly effective for removing electrical Shift Artifact.

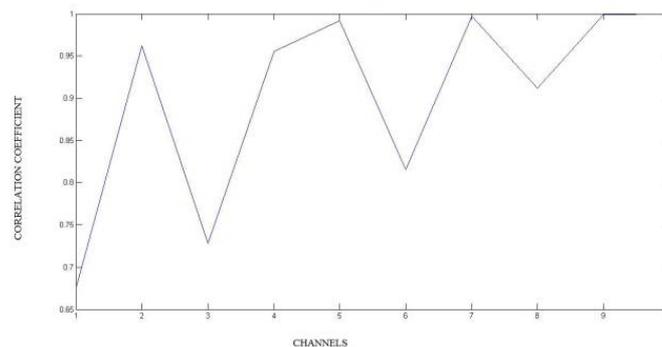


Figure11: Correlation Coefficients for Technique-2 (ICA + HC)

This technique gives a satisfactory performance for all the channels affected by artifacts but performs poorly in case of Electrical Shift artifact.

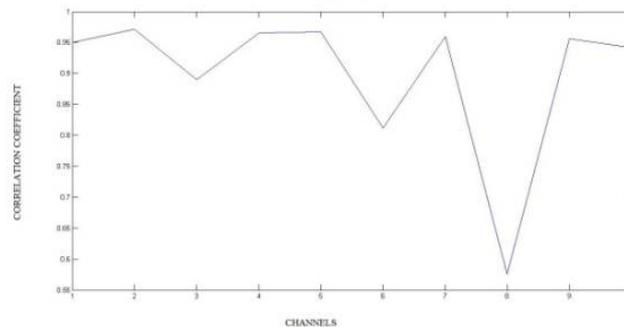


Figure 12: Correlation Coefficients for Technique-3 (DWT + ICA + HC)

As can be seen from the above figure it gives a good performance for channel -1 or the channel affected by eye blink artifact and performs the best in case of removing linear trend artifact. However, it performs very poorly in case of removing Electrical Shift artifact.

VI. ANALYSIS

All the three techniques perform differently for different types of artifacts as can be seen from the Table 2. The original dataset consisted of EEG recordings of 10 seconds from all the 10 channels. Artifacts were added to 4 channels-1, 3, 6 and 8. The performance of these techniques for different artifacts have been summarised as follows.

Eye Blink Artifact: It was added to Channel-1 with amplitude of 120 units. The decreasing order of the efficiency of techniques in eliminating this is:

Technique-2 > Technique-3 > Technique-1

Muscle Artifact: It was added to Channel-3. The techniques were not as effective in this case as they were in removing eye blink artifact. The decreasing order of the effectiveness of the techniques in removing it is:

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 7, July 2016

Technique-3 > Technique-1 > Technique-2

Linear Trend Artifact: This artifact occurs when the electrodes have not been properly adjusted on the scalp and they tend to slip. It was added to Channel-6. There is good amount of artifact rejection in this case. The decreasing order of efficiency of techniques in removing this is:

Technique-1 > Technique-2 > Technique-3

Electrical Shift Artifact: This artifact may occur due to improper setting of the recording apparatus. It was added to channel-8. Technique-1 was the most effective in removing this. The decreasing order of the effectiveness of techniques in this case:

Technique-1 > Technique-2 > Technique-3

Thus, from the above analysis, it can be concluded that while technique-1 was the most effective in removing non-physiological artifact signals, technique-3 was the most effective in removing physiological artifacts. Technique-2 performed moderately in both types of artifacts.

Table 2: Correlation Coefficient

CHANNEL NO.	CHANNEL NAME	TECHNIQUE-1	TECHNIQUE-2	TECHNIQUE-3
1	Fz	0.8027	0.9639	0.9593
2	FC5	0.9032	0.9993	0.9829
3	FC6	0.8328	0.7420	0.8660
4	Cz	0.9324	0.9994	0.9706
5	P3	1.000	0.9972	0.9689
6	Pz	0.9504	0.8504	0.8124
7	P4	0.9715	0.9967	0.9684
8	PO7	0.9180	0.6140	0.5732
9	Oz	1.000	0.9926	0.9631
10	PO8	0.9881	0.9809	0.9498

VII. CONCLUSION AND FUTURE SCOPE

EEG records the electrical activity occurring inside the brain, due to communication that takes place between neurons that constitute the brain. It is a vital bio-potential that reflects variations in brain's elaborate and complex internal activities and hence can provide valuable understanding of the brain's internal working. It is found to be very useful in spotting neural ailments and also in the development of Human Machine Interfaces (HMI). However, EEG is highly vulnerable to being corrupted by non-cognitive signals whether biological or non-biological, called artifacts, which can considerably affect the end performance of an EEG based system. EEG processing has gained impetus in recent years by the scientific community, mostly for developing new age HMIs, which use the brain waves to directly control external hardware. However, the presence of artifacts in EEG is a major apprehension, which calls for efficient and automatic removal of these noise signals. Hence automatic removal of these artifacts becomes a necessity to make future HMIs. Several methods have been proposed in literature to remove the artifacts- both automatic as well as manual. In this dissertation three such techniques are studied- ICA, DWT and Hierarchical Clustering. Thereafter a new system based on the combined use of the above techniques is proposed. The implementation was done in MATLAB and EEGLAB. The output is tested by performing correlation between the original and processed signals and

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 7, July 2016

encouraging results are obtained. Technique-1, which was based on ICA and DWT, is very effective in removing non-physiological noise signals while the technique-2, which was based on ICA and Hierarchical Clustering, performs moderately well for both types of noise signals. The proposed technique is found to be very effective in eliminating biological artifacts i.e. eye blinks and muscle artifacts and performs moderately well for non-biological artifacts. The proposed technique is found to remove eye blinks very efficiently and hence can be employed in such applications where eye blink is a major disturbance.

Future Scope: The proposed technique is found to be very effective in eliminating biological origin artifacts. An effort can be made to further improvise this technique for a dedicated removal of biological artifacts via the use of neural networks. NNs can be trained to detect electrodes more prone to being affected by such biological artifacts and then clean those using the discussed technique. This technique holds a very promising future to be employed in such BCIs or HMIs which are more prone to being affected by eye blinks and other biological artifacts. A few such applications are Virtual Reality navigation systems, eye-tracking technology etc. It can also be used in psychological studies which involve the patient to concentrate on the screen for longer times. It can also prove very efficient for psychological studies involving children as they are more prone to blinking frequently thereby generating a lot of disturbance in the EEG signal. Thus, it can be used to make psychological studies more efficient.

REFERENCES

- [1] Gramann Klaus Daniel P. Ferris, Joseph Gwin and Scott Makeig. "Review Imaging natural cognition in action" ELSEVIER International Journal of Psychophysiology, Vol 91, 2014, pp. 22–29.
- [2] Teplan M. "Fundamentals of EEG Measurements" Measurement Science Review, Volume 2, Section 2, 2002, pp. 1-11.
- [3] Mammone Nadia, Fabio La Foresta and Francesco Carlo Morabito, "Automatic Artifact Rejection from Multichannel Scalp EEG by Wavelet ICA", *IEEE Sensors Journal*, Vol. 12, No. 3, DOI: 10.1109/JSEN.2011.2115236, 2012 March, pp. 533-544.
- [4] Inuso Guiseppina, Fabio La foresta, Nadia Mammone and Francesco carlo Morabito. "Wavelet-ICA Methodology for efficient artifact removal from Electroencephalographic recordings" *Proceedings of International Joint Conference on Neural Networks, USA, 2007*.
- [5] Roy Vandana and Dr. Shukla Shailja "Automatic Removal of Artifacts from EEG Signal Based on Spatially Constrained ICA Using Daubechies Wavelet", *I.J. Modern Education and Computer Science*, DOI: 10.5815/ijmecs.2014.07.05, 2014 July, pp. 31-39.
- [6] Geetha G. and Geethalakshmi S.N., "Artifact Removal from EEG using Spatially Constrained Independent Component Analysis and Wavelet Denoising with Otsu's Thresholding Technique", *International Conference on Communication Technology and System Design*, 2011, pp. 1064-1071.
- [7] Zhang Chi, Li Tong, Ying Zeng, Jingfang Jiang, Haibing Bu. "Automatic Artifact Removal from Electroencephalogram Data based on Apriori Artifact Information", *Hindawi Publishing Corporation BioMed Research International*, Volume 2015, Article ID 720450, 2015 March, pp. 1-8.
- [8] Patidar Udit and George Zouridakis, "A Hybrid Algorithm for Artifact Rejection in EEG Recordings Based on Iterative ICA and Fuzzy Clustering", *30th Annual International IEEE EMBS Conference, Vancouver, British Columbia, Canada, 2008 August 20-24*, pp. 50-53.
- [9] Meghdad Ashtiyani, Saeed Asadi, Parmida Moradi Birgani. "ICA-Based EEG Classification Using Fuzzy C-mean Algorithm" *Unpublished*.
- [10] Hongzhi Qi, Yuhuan Zhu, Dong Ming, Baikun Wan, Yizhong Wang, Rui Zhang, "Independent Component Automatic Clustering and Its Application on Multi-Trails Imaginary Hand Movement Related EEG", *VECIMS 2009 - International Conference on Virtual Environments, Human-Computer Interfaces and Measurements Systems, 2009*.
- [11] Zou Yuan, Nathan Viswam and Jafari Roozbeh. "Automatic Identification of Artifact-Related Independent Components for Artifact Removal In EEG Recordings", *IEEE Journal of Biomedical and Health Informatics*, 2014, pp. 1-9
- [12] Zou Yuan, John Hart, Jr. and Roozbeh Jafari, "Automatic EEG Artifact Removal Based on ICA and Hierarchical Clustering" *ICASSP*, 2012, pp. 649-652.
- [13] Zou Yuan, Nathan Viswam and Jafari Roozbeh, "Automatic Removal of EEG Artifacts using Electrode-Scalp Impedance" *IEEE International Conference on Acoustic, Speech and Signal Processing (ICASSP)*, 2014, pp. 2073-2077
- [14] Milanese M., C.J. James, N. Martini, A. Gemignani, D. Menicucci, B. Ghelarducci, L. Landini, "Late Positive Event-Related Potentials Enhancement through Independent Component Analysis Clustering", *Unpublished*.
- [15] Lekshmi S.S., V.Selvam, M.Pallikonda Rajasekaran. 2014. EEG Signal Classification using Principal Component Analysis And Wavelet Transform with Neural Network, *International Conference on Communication and Signal Processing*, India, 2014 April 3-5, pp. 687-690.
- [16] Vigário Ricardo, Jaakko Särelä, Veikko Jousmäki, Matti Hämäläinen, and Erkki Oja, Independent Component Approach to the Analysis of EEG and MEG Recordings, *IEEE Transactions on Biomedical Engineering*, Vol. 47, No. 5, 2000 May, pp. 589-593.
- [17] Vázquez R. Romo, H. Vélez-Pérez, R. Ranta, V. Louis Dorr, D. Maquin, L. Maillard. 2012. Blind Source Separation, Wavelet Denoising and Discriminant Analysis for EEG Artefacts and Noise Cancelling, *Elsevier Biomedical Signal Processing and Control*, DOI:10.1016/j.bspc.2011.06.005, 2012, pp. 389-400.
- [18] Teich Patrick. 2015. Master's Thesis: Designing a Brain Computer Interface Using an Affordable EEG Headset, Master's Thesis, Freie Universität, Berlin, 2015, January 3



ISSN(Online) : 2319-8753
ISSN (Print) : 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 7, July 2016

- [19] Scott Makeig and Julie Onton, ERP Features and EEG Dynamics: An ICA Perspective. Oxford handbook of event related PotentialComponents, Oxford University Press, 2011.
- [20] AnttiSavelainen, An Introduction to EEG Artifacts, *Mat-2.4108 Independent Research Project in applied mathematics*, February 20, 2010, pp. 1-22.
- [21] Arnaud Delorme, Scott Makeig, EEGLAB: an open source toolbox for analysis of single-trial EEG dynamics including independent component analysis, *Journal of Neuroscience Methods, Elsevier B.V*, Vol 164, 2004, pp. 1-14.